National
Qualifications
2021 ASSESSMENT RESOURCE

X847/77/11

Mathematics
Paper 1 (Non-calculator)

Duration — 1 hour

Total marks — 35

Attempt ALL questions.

You must NOT use a calculator.

To earn full marks you must show your working in your answers.

State the units for your answer where appropriate.

You will not earn marks for answers obtained by readings from scale drawings.

Write your answers clearly in the spaces provided in the answer booklet. The size of the space provided for an answer is not an indication of how much to write. You do not need to use all the space.

Additional space for answers is provided at the end of the answer booklet. If you use this space you must clearly identify the question number you are attempting.

Use **blue** or **black** ink.

Before leaving the examination room you must give your answer booklet to the Invigilator; if you do not, you may lose all the marks for this paper.

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FORMULAE LIST

Standard derivatives

$f(x)$	$f'(x)$
$\sin^{-1} x$	$\frac{1}{\sqrt{1-x^2}}$
$\cos^{-1} x$	$-\frac{1}{\sqrt{1-x^2}}$
$\tan^{-1} x$	$\frac{1}{1+x^2}$
$\tan x$	$\sec^2 x$
$\cot x$	$-\operatorname{cosec}^2 x$
$\sec x$	$\sec x \tan x$
$\operatorname{cosec} x$	$-\operatorname{cosec} x \cot x$
$\ln x$	$\frac{1}{x}$
e^x	e^x

Standard integrals

$f(x)$	$\int f(x)\,dx$
$\sec^2(ax)$	$\frac{1}{a}\tan(ax)+c$
$\frac{1}{\sqrt{a^2-x^2}}$	$\sin^{-1}\left(\frac{x}{a}\right)+c$
$\frac{1}{a^2+x^2}$	$\frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right)+c$
$\frac{1}{x}$	$\ln\lvert x\rvert+c$
e^{ax}	$\frac{1}{a}e^{ax}+c$

Summations

(Arithmetic series) $$S_n = \frac{1}{2}n\left[2a+(n-1)d\right]$$

(Geometric series) $$S_n = \frac{a\left(1-r^n\right)}{1-r},\ r \neq 1$$

$$\sum_{r=1}^{n} r = \frac{n(n+1)}{2},\quad \sum_{r=1}^{n} r^2 = \frac{n(n+1)(2n+1)}{6},\quad \sum_{r=1}^{n} r^3 = \frac{n^2(n+1)^2}{4}$$

Binomial theorem

$$(a+b)^n = \sum_{r=0}^{n}\binom{n}{r}a^{n-r}b^r \quad \text{where} \quad \binom{n}{r} = {}^nC_r = \frac{n!}{r!(n-r)!}$$

Maclaurin expansion

$$f(x) = f(0)+f'(0)x+\frac{f''(0)x^2}{2!}+\frac{f'''(0)x^3}{3!}+\frac{f^{iv}(0)x^4}{4!}+\ldots$$

FORMULAE LIST (continued)

De Moivre's theorem

$$\left[r(\cos\theta + i\sin\theta)\right]^n = r^n\left(\cos n\theta + i\sin n\theta\right)$$

Vector product

$$\begin{aligned}\mathbf{a}\times\mathbf{b} &= |\mathbf{a}||\mathbf{b}|\sin\theta\,\hat{\mathbf{n}}\\ &= \begin{vmatrix}\mathbf{i} & \mathbf{j} & \mathbf{k}\\ a_1 & a_2 & a_3\\ b_1 & b_2 & b_3\end{vmatrix} = \mathbf{i}\begin{vmatrix}a_2 & a_3\\ b_2 & b_3\end{vmatrix} - \mathbf{j}\begin{vmatrix}a_1 & a_3\\ b_1 & b_3\end{vmatrix} + \mathbf{k}\begin{vmatrix}a_1 & a_2\\ b_1 & b_2\end{vmatrix}\end{aligned}$$

Matrix transformation

Anti-clockwise rotation through an angle, θ, about the origin, $\begin{pmatrix}\cos\theta & -\sin\theta\\ \sin\theta & \cos\theta\end{pmatrix}$

[Turn over

MARKS

Total marks — 35

Attempt ALL questions

1. (a) Differentiate $y = x^3 e^{5x}$. **2**

(b) Given $y = \dfrac{\tan x}{x^6 + 1}$, find $\dfrac{dy}{dx}$. **2**

2. Matrices A and B are defined as follows

$$A = \begin{pmatrix} -2 & 4 \\ -3 & 7 \end{pmatrix}, \quad B = \begin{pmatrix} 4 & 0 \\ 2 & 3 \\ -2 & 1 \end{pmatrix}.$$

Find

(a) AB', where B' is the transpose of B. **2**

(b) A^{-1}. **2**

3. Use the substitution $u = \sin\theta$ to find $\int \cos\theta \sin^3\theta \, d\theta$.

Write your answer in terms of θ. **2**

4. A system of equations is given by

$$\begin{aligned} x + 2y + z &= 5 \\ 3x - y + 2z &= 4 \\ -2x + 3y + \lambda z &= -8 \end{aligned}$$

where $\lambda \in \mathbb{R}$.

Use Gaussian elimination to determine the value of λ for which this system of equations has no solution. **4**

5. A solid is formed by rotating the curve with equation $y = 2\sqrt{x}$ between $x = 3$ and $x = 5$ through 2π radians about the x-axis.

Calculate the exact value of the volume of this solid. **2**

MARKS

6. The velocity, v m s^{-1}, of a particle after t seconds is given by $v = 3t^2 - e^{-2t}$.

At time $t = 0$ the displacement of the particle is zero.

(a) Find an expression for the displacement of the particle. **2**

(b) Calculate the acceleration of the particle when $t = 0$. **2**

7. A function is defined on a suitable domain by $f(x) = \dfrac{x^2}{x-2}$.

(a) For the graph of $y = f(x)$

(i) state the equation of the vertical asymptote **1**

(ii) find the equation of the non-vertical asymptote.

Justify your answer. **2**

The turning points on the graph are (0, 0) and (4, 8).

There are no other stationary points.

(b) On the diagram provided, sketch the graph of $y = f(x)$. **1**

(c) (i) On the diagram provided, sketch the graph of $y = |f(x)|$.

Show all asymptotes. **1**

(ii) State the values of k for which $|f(x)| = k$ has exactly two distinct solutions. **1**

8. Find the particular solution of the differential equation

$$\frac{d^2y}{dx^2} + \frac{dy}{dx} - 6y = 35e^{2x}$$

given $y = 5$ and $\dfrac{dy}{dx} = 12$ when $x = 0$. **9**

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